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Engineering Notes

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Vibration Control of a Flexible Beam Using Shape Memory Alloy Actuators

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Introduction

THE shape memory alloy (SMA) actuators produce relatively large control forces compared to other actuating materials such as piezoceramics. In addition, design simplicity for control mechanism, high possibility of miniaturization, and low power consumption are salient properties of the SMA actuators. Accordingly, this class of actuators has been proven to be successful in vibration control of flexible structures. Baz et al.[1] applied the SMA actuators to active vibration control of a flexible cantilevered beam. They derived a control law on the basis of the displacement position of the flexible beam. This logic can be called a constant amplitude controller, which provides constant magnitude control input during the whole control action. Favorable vibration suppression was accomplished, but unbalanced response was observed with respect to the neutral axis of the beam. Furthermore, undesirable chattering also occurred in the settled phase. This is attributable to excessive supply of control inputs for relatively small levels of oscillations.

New control strategies to overcome these drawbacks are presented. Three types of control schemes are designed: constant-amplitude controller (CAC), proportional-amplitude controller (PAC), and sliding-mode controller (SMC). The proposed controllers are implemented and evaluated by investigating the level of control suppression in transient vibration.

System Modeling

The arrangement of the experimental apparatus for the vibration control is shown in Fig. 1. Two SMA actuators are installed on opposite sides of the cantilevered beam with a certain inclined angle. The beam-actuators system model can be formulated through a typical finite element method.[1] We consider controlling only the first bending mode of the beam (y direction in the figure), as was done previously.[1] Thus, the control system can be expressed in the transformed modal coordinate (η) as

$$\ddot{\eta}(t) + 2\zeta\omega\dot{\eta}(t) + \omega^2\eta(t) = f(t) \tag{1}$$

where $f(t)$ represents the actuating force and ζ and ω denote the first mode damping ratio and natural frequency, respectively.

Two inherent dynamic characteristics of the SMA actuator $f(t)$ should first be identified to achieve successful implementation. Figure 2 presents a unit step response of the employed SMA actuator. We can assume that the force resulting from thermal expansion is negligible compared to the transformation force. Furthermore, we can assume that the transformation force is generated and recovered exponentially with time.[1] With these assumptions, the actuator force $f(t)$ is given by

$$f(t) = \begin{cases} 0, & 0 < t \le t_1 \\ f_{\max}\left[1 - \exp\dfrac{-(t-t_1)}{\tau_h}\right], & t_1 < t \le t_2 \\ f_{\max}, & t_2 < t \le t_3 \\ f_{\max}\left[\exp\dfrac{-(t-t_3)}{\tau_c}\right], & t_3 < t \le t_4 \\ 0, & t > t_4 \end{cases} \tag{2}$$

where $f_{\max}$ is the maximum force that the actuator can develop and τ_h and τ_c represent time constants at the heating and cooling stages, respectively.

The control system (1) needs to be modified to account for physical input that is current. Thus, the relationship between input current $i(t)$ and output force $f(t)$ should be established for the heating stage. This can be obtained from the second equation in Eqs. (2), and hence the control system (1) is rewritten as

$$\ddot{\eta}(t) + 2\zeta\omega\dot{\eta}(t) + \omega^2\eta(t) = \hat{k}_b i(t) \tag{3}$$

Here $\hat{k}$ is the input influence coefficient.

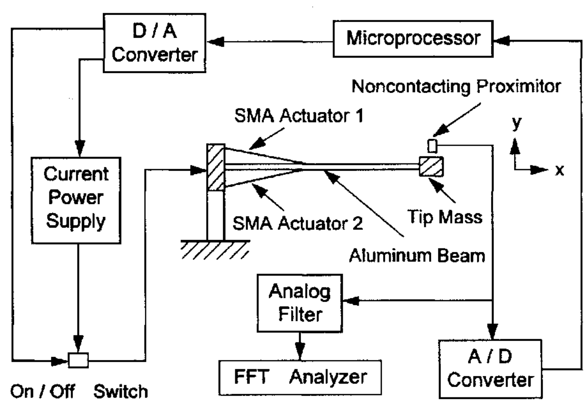

Fig. 1 Experimental apparatus for vibration control.

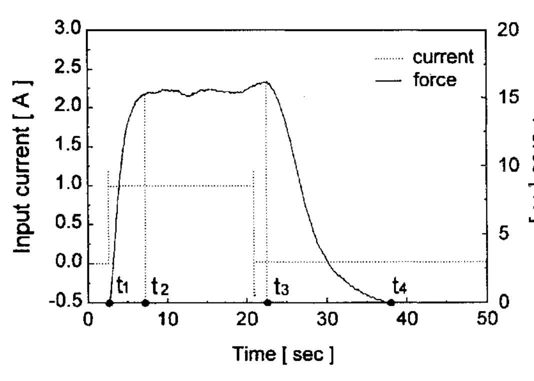

Fig. 2 SMA actuator characteristics.

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Controller Design

CAC and PAC

The impending issue is to suppress structural vibration of the cantilevered beam by applying control current $i(t)$ to the SMA actuators. The principle of the CAC algorithm is based on the physical phenomenon of the vibration system. The actuator needs to be energized to bring the system tc its equilibrium state (original position). Thus, actuator 2 in Fig. 1 should be activated when an external disturbance acts on the beam, deflecting it in the positive y direction, while actuator 1 is off. This control logic can be stated by the following mathematical expression:

$$i(t) = -k_1 \mathrm{sgn}[y_e(t)] \tag{4}$$

where k_1 is the feedback gain, $y_e(t)$ is the tip deflection, and sgn is the nonlinear signum function.

It is clear that the magnitude of the control input current in the CAC algorithm remains constant throughout the control action. This may cause the flexible structure to produce undesirable chattering in the settled phase. This is attributed to an excessive supply of the control input for small levels of oscillations. This drawback can be resolved by employing time-varying control input the magnitude of which is proportionally adjusted with respect to the oscillation magnitude. The mathematical expression of this control logic (call it the PAC, for convenience) can be given by

$$\begin{aligned} i(t) &= -k_1 \mathrm{sgn}[y_e(t)], & y_{e1} < y_e(t) \le y_0 \\ i(t) &= -k_1 y_e(t) \mathrm{sgn}[y_e(t)], & y_e(t) \le y_{e1} \end{aligned} \tag{5}$$

Here y_0 is the initial tip deflection and y_{e1} is the tip deflection at the time at which the control gain $k_1 y_e(t)$ begins to activate. It is obvious that the level of y_{e1} has a decisive effect on the PAC algorithm's effectiveness in attenuating the chattering. Here, the level of y_{e1} is experimentally determined by observing the existence of the chattering with respect to the input magnitude.

SMC

One salient feature of the SMC is the sliding motion of the state on the sliding surface.[2] During this sliding motion, the system has invariance properties, yielding robust motion to uncertain parameters. In addition, the simplicity of design and implementation attracts numerous applications. From these aspects, we adopt the SMC for vibration control of the flexible beam using the SMA actuators. To establish the problem formulation, the system equation (3) is rewritten in the state-space form as

$$\begin{gathered} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\omega^2 x_1(t) - 2\zeta\omega x_2(t) + bu(t) \\ x(t_0) = x_0 \end{gathered} \tag{6}$$

where $[x_1\ x_2]^T = [\eta\ \dot{\eta}]^T$, $b = \hat{k}_b$, $u(t) = i(t)$, and x_0 are the initial conditions given at initial time t_0. The control problem is to damp out $x_i(t)$ for arbitrary initial conditions. Thus, we define a sliding surface as

$$\sigma(t) = cx_1(t) + x_2(t), \qquad c > 0 \tag{7}$$

We see that the state trajectory $x_i(t)$ becomes zero for any initial conditions, provided that there exists a control $u(t)$ so as to cause the state trajectory to slide along the surface defined by Eq. (7). This can be achieved by satisfying the sliding condition

$$\sigma(t)\dot{\sigma}(t) < 0 \tag{8}$$

We construct such a discontinuous control $u(t)$ from the concept of equivalent control as

$$u(t) = \frac{-\left[-\omega^2 x_1(t) + (c - 2\zeta\omega)x_2(t) + k \cdot \mathrm{sgn}(\sigma)\right]}{b} \tag{9}$$

where $k(>0)$ is the feedback gain.

We can obtain the optimal value of c with given initial states and feedback gain k. The application of the optimal value of c provides much better control performance in the sense of convergence speed than when we choose an arbitrary value of c without considering the initial states and feedback gain k. The optimal value of c is determined by minimizing the following performance index J (Ref. 2):

$$\begin{aligned} J &= \int_0^\infty \left[t^2 x_1^2(t)\right] \mathrm{d}t \\ &= \int_0^{t_s} \left[t^2 x_1^2(t)\right] \mathrm{d}t + \int_{t_s}^\infty \left[t^2 x_1^2(t)\right] \mathrm{d}t \end{aligned} \tag{10}$$

where t_s is the time at which the sliding mode begins.

On the other hand, the implementation of the discontinuous control law (9) causes chattering in the control history that is an impediment to the SMC. Thus, in the experimental realization the controller (9) is to be modified by replacing the signum function with the saturation function.

Results and Discussion

The employed aluminum beam is 380 mm long, 22.3 mm wide, and 1.13 mm thick. The diameter of the SMA actuator wire is 0.36 mm, and its transformation temperature is 38°C. The actuators were installed on the beam at a distance of 120 mm from the clamped position with an angle of 22.6 deg. The mass of 27.6 g was attached at the free end of the cantilevered beam. In Fig. 1, A/D and D/A converters have 12 bits. The controllers were implemented using a personal computer (IBM 486) with a sampling rate of 1000 Hz.

Figure 3 presents measured transient-vibration control responses. The transient vibrational response was generated by imposing an initial tip deflection of 5.3 mm. It is clearly observed that the oscillation in the absence of control current only decays with the inherently existing damping. It is distilled from the open-loop response that the first-mode natural frequency of the structure is 4.1 Hz. The response with the CAC ($k_1 = 1.0$) was almost suppressed within 5.5 s, but shows undesirable chattering in the settled phase. As

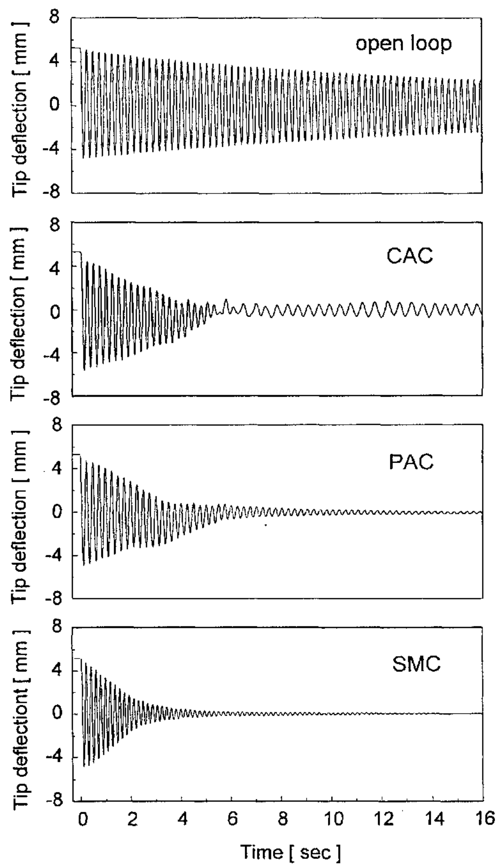

Fig. 3 Vibration control responses.

mentioned earlier, this chattering arises from the excessive supply of control input to relatively small oscillations. This chattering is reduced for the lower levels of control current by trading off decay time.

By employing the PAC, the chattering was completely eliminated without deteriorating the decay time. This implies that the PAC algorithm produces relatively small adverse control force associated with the input current in the settled phase. The threshold tip deflection (y_{e1}), which decides the activation time of the PAC, was 0.73 mm in this realization. On the other hand, for the implementation of the SMC the discontinuous feedback gain $k = 0.7$ was employed. With the feedback gain and imposed initial conditions, the optimal value of $c = 6.9$ was determined to minimize the performance index J given by Eq. (10). It is seen that the tip deflection was favorably suppressed without the unbalancing phenomenon that occurred in the CAC and the PAC. Note that the control characteristics of the SMC depend on the design parameters k and c.

Conclusion

An active vibration control utilizing the SMA actuators has been undertaken to suppress the bending vibrations of a cantilevered beam. A modified CAC was proposed and successfully implemented to eliminate undesirable chattering in the settled phase. In addition, superior control performance was demonstrated by employing the SMC. The control effects attributable to the number of actuators and the location of the actuators are to be studied in the future.

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Near-Optimal Operation of Dual-Fuel Launch Vehicles

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Introduction

CURRENT studies of single-stage-to-orbit (SSTO) launch vehicles are focused on all-rocket propulsion systems.[1,2] One option for such vehicles is the use of dual fuel [liquid hydrocarbon and liquid hydrogen (LH_2)] for a portion of the mission.[3–6] Compared with LH_2, hydrocarbon fuel has higher density and produces higher thrust to weight but has lower specific impulse. These advantages of hydrocarbon fuel are important early in the ascent trajectory, where vehicle weight is high, and its use may be expected to lead to reduced vehicle size and weight. Because LH_2 is also typically needed for cooling purposes, in the early portion of the trajectory both fuels usually must be burned simultaneously. Later in the ascent, when vehicle weight is lower and thrust requirements are less, specific impulse is the key parameter, indicating single-fuel LH_2 use.

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Two recent papers[5,6] have considered the optimization of dual-fuel SSTO vehicles. Included in the studies was a determination of M_{tr}^*, the Mach number at which to transition from dual-fuel mode to LH_2 operation to minimize vehicle empty weight. Both of these references treat M_{tr} as an external design parameter, which must be optimized by iterative design evaluations.

In this Note, a guidance algorithm is developed that determines whether dual-fuel or single-fuel operation is superior as an integral part of the trajectory integration. This approach saves a substantial number of iterations of a computer design code by reducing the number of design variables and hence the number of design iterations required in a vehicle optimization study. Further, the guidance law will be directly usable as part of a real-time, onboard propulsion control system.

The basis of the guidance law is the energy-state dynamic model. The key idea is to introduce the total mechanical energy as a state variable and then to neglect all other dynamics. This results in a function optimization problem. When flight-path optimization is done with this model, simple rules for the optimal path and for the optimal operation of propulsion system are obtained. This dynamic model has been used successfully many times to obtain effective guidance laws for a wide variety of aircraft and missions (see Ref. 7 and the references therein for a review of this work). The energy-state approach is particularly suitable for launch vehicles because efficient energy accumulation (or equivalently maximizing total ΔV) is the primary trajectory optimization goal.

In a series of papers,[7–9] we have used energy-state methods to develop algorithms for ascent trajectory optimization and optimal operation of single-fuel multimode propulsion systems. In particular, the operation of propulsion systems with two separate engines, airbreathing and rocket, was investigated.[9] The present Note extends those methods to the dual-fuel case. The main goal is to determine M_{tr}^* and to investigate optimal trajectories.

In the numerical results, vehicle performance is computed using the NASA Ames Research Center code HAVOC. HAVOC integrates geometry, aerodynamics, propulsion, structures, weights, and other computations to produce point designs for a wide variety of launch vehicles. It is capable of iteratively determining closed vehicles, that is, designs that meet specified payload mass and volume requirements for a specified mission. Although the trajectory guidance law is based on the energy-state model, the trajectory integration in HAVOC uses a point-mass model, including the effects of the Earth's rotation, the Earth's curvature, and variable gravity.

Optimization Function

The energy-state model is obtained by using the total mechanical energy per unit weight as the state variable[7–9]:

$$\dot{E} = P \tag{1}$$

$$\dot{W} = -(T/I_{SP}) \tag{2}$$

where

$$E = [hR/(R+h)] + (1/2g)V^2 \tag{3}$$

and

$$P = (V/W)(T_v - D) \tag{4}$$

and where the drag is evaluated at the lift required for equilibrium of forces perpendicular to the flight path. In these equations, E, P, W, T, I_{SP}, h, R, V, g, T_v, and D are energy per unit weight, specific excess power, weight, thrust, specific impulse, altitude, the Earth's radius, gravitational acceleration at the Earth's surface, component of thrust along the velocity vector, and drag, respectively.

For an SSTO mission, what is desired is a trajectory that gives the minimum-empty-weight vehicle to put a given payload mass and volume in orbit. Because the density of liquid hydrogen is low, the sensitivity of perturbations in volume needs to be taken into consideration as well as mass sensitivity, and it is therefore necessary